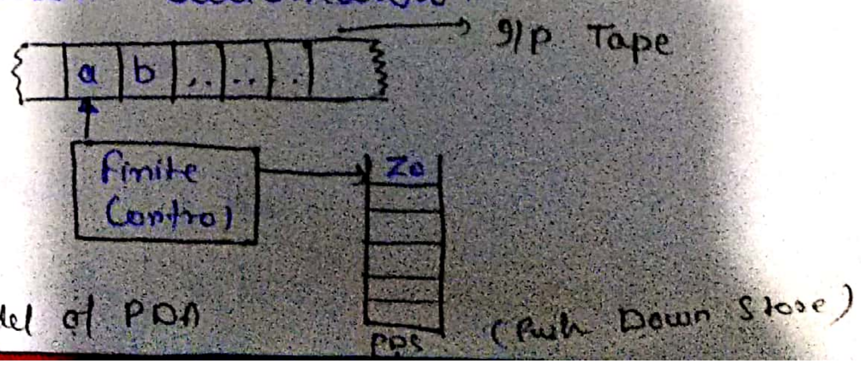
Unit-4

Push Down Automata

Let us consider $L = a^n b^n \mid n \geq 1$ this is a CFL but not regular $(S \to aSb \mid ab)$. A finite automaton cannot accept L i.e the string of the form $a^n b^n$ as it has to remember the no. of a's in a string & so it will require in finite number of states. This difficulty can be avoided by adding an auxiliary memory in the form of stack. The 'a' in the given string are added to the stack when the symbol 'b' is encountered in the input string and 'a' is removed from the stack thus the matching of no. of a's & no. of b's is accomplished. This type of arrangement where a finite automaton has a stack is called a pushed down automaton.



A model of PDA

A PDA components are as :-

It has a read only input tape, input alphabet, finite state control, a set of final states & an initial state as in the case of finite automaton.

In addition to these it has a stack called the PDS. It is a read/write push down store as we add elements to PDS or remove from PDS.

A PDA consist of 7 tuples

$A = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ as :-

(i) The finite non-empty set of states denoted by Q.

(ii) The finite non-empty set of input symbols denoted by Σ.

(iii) The finite non-empty set of push down symbols denoted by Γ.

(iv) $q_0 \in Q$ is the initial state.

(v) $Z_0 \in \Gamma$ is called an initial pushed down store symbol.

(vi) $F \subseteq Q$ is the final state.

(vii) δ is the transition function mapping from $Q \times (\Sigma \cup \{\Lambda\}) \times \Gamma$ into $Q \times \Gamma^*$

Ques. Design a PDA for the language $L = a^n b^n | n \geq 1$

Sol^n. Step-1 Create Grammar $S \rightarrow aSb | ab$

Step 2. Convert into GNF

$S \rightarrow aSA | aA$

$A \rightarrow b$

Step 3 - Transition Rule

$\delta(q_0, \wedge, z_0) = (q_1, Sz_0)$

Initial state

$\delta(q_1, a, S) = \{(q_1, SA), (q_1, A)\}$

$\delta(q_1, b, A) = (q_1, \wedge)$

Final state $\delta(q_1, \wedge, z_0) = (q_f, \wedge)$

Ques $L = a^n b^n | n \geq 1$ Generate ID for aabb

$(q_0, \wedge aabb, z_0) \vdash (q_1, aabb, Sz_0) \vdash$

$(q_1, abb, SAz_0) \vdash (q_1, bb, AAz_0) \vdash$

$(q_1, b, Az_0) \vdash (q_1, \wedge, z_0) \vdash (q_f, \wedge)$

Ques. Design a PDA for $S \rightarrow aSbb | a$ & generate a ID for aaabbbb

Sol^n- Step 1:- Convert it into GNF

$S \rightarrow aSA | a$

$A \rightarrow bB$

$B \rightarrow b$

Step 2- First we convert the given grammar into GNF. The GNF of the above grammar is

$S \to aSA \mid a$

$A \to bB$

$B \to b$

Let us consider a PDA $(Q, \Sigma, T, q_0, \delta, F, Z_0)$ which is equivalent to the given grammar,

Here $Q = \{q_0, q_1, q_f\}$, $\Sigma = \{a, b\}$, $T = \{Z_0, S, A, B\}$ & q_0 is the initial state, Z_0 is the initial push down store symbol & q_f is the final state, δ is the transition function mapping from $Q \times (\Sigma \cup \{\Lambda\}) \times T$ into $Q \times T^*$ define below.

Step-2

$$\delta(q_0, \Lambda, Z_0) = (q_1, SZ_0)$$

$$(q_1, a, S) = \{(q_1, SA), (q_1, \Lambda)\}$$

$$\delta(q_1, b, A) = (q_1, B)$$

$$\delta(q_1, b, B) = (q_1, \Lambda)$$

$$\delta(q_1, \Lambda, Z_0) = (q_f, \Lambda)$$

Step 3.

ID:-

$(q_0, \Lambda aaabbbb, Z_0) \vdash (q_1, aaabbbb, SZ_0) \vdash (q_1, abbbb, SAAZ_0) \vdash$

$\vdash (q_1, aabbbb, SAZ_0) \vdash (q_1, abbbb, SAAZ_0) \vdash (q_1, bb, AZ_0)$

$(q_1, bbbb, AAZ_0) \vdash (q_1, bbb, BAZ_0) \vdash (q_1, bb, AZ_0)$

$\vdash (q_1, b, BZ_0) \vdash (q_1, \Lambda, Z_0) \vdash (q_f, \Lambda)$

Ques Construct a PDA A equivalent to the following CFG $S \to 0BB$

$B \to 0S \mid 1S \mid 0$

Test whether 010^4 is in $N(A)$

Soln.- Step 1- Done

Step 2 Done

Step 3. $\delta(q_0, \Lambda, Z_0) = (q_1, SZ_0)$

$\delta(q_1, 0, S) = (q_1, BB\,Z_0)$

$\delta(q_1, 0, B) = \{(q_1, S), (q_1, \Lambda)\}$

$\delta(q_1, 1, B) = (q_1, S)$

$\delta(q_1, \Lambda, Z_0) = (q_f, \Lambda)$

Step 4. Generate ID for 010000

$(q_0, \Lambda 010000, Z_0) \vdash (q_1, 010000, SZ_0) \vdash$

$(q_1, 10000, BBZ_0) \vdash (q_1, 0000, SBZ_0) \vdash$

$(q_1, 000, BBBZ_0) \vdash (q_1, 00, BBZ_0) \vdash$

$(q_1, 0, BZ_0) \vdash (q_1, \Lambda, Z_0) \vdash (q_f, \Lambda)$

PDA to CFG :-

Let $M = (Q, \Sigma, T, \delta, q_0, Z_0, F)$ be a PDA excepting L construct a grammar $G = (V_N, \Sigma, P, S)$ such that $L(G) = N(M)$

(i) Let $V_N = \{S\} \cup \{[q, A, p] \mid q, p \in Q, A \in T\}$

(ii) $|V_N| = k^2 \times m + 1$ (k-states, m - PDB symbols)

$T = \Sigma$

(iii) P: $S \rightarrow [q_0, Z_0, p]$ for each p in Q

if $\delta(q, x, A)$ contain $(p, B_1, B_2 \ldots B_m)$ $m \neq 0$

$$[q, A, q_4] \rightarrow x[p, B_1, q_2][q_2, B_2, q_3] \ldots [q_3, B_m, q_4]$$

$\Rightarrow$ $\delta(q, x, A) = (p, \wedge)$ $m = 0$

production - $[q, A, p] \rightarrow x$

Ques - $M = (\{q_0, q_1\}, \{0, 1\}, \{Z_0, X\}, \delta, q_0, Z_0, \phi)$

1. $\delta(q_0, 0, Z_0) = (q_0, XZ)$
2. $\delta(q_0, 0, X) = (q_0, XX)$
3. $\delta(q_0, 1, X) = (q_1, \wedge)$
4. $\delta(q_1, 1, X) = (q_1, \wedge)$
5. $\delta(q_1, \wedge, X) = (q_1, \wedge)$
6. $\delta(q_1, \wedge, Z_0) = (q_1, \wedge)$

1- $S, [q_0, Z_0, q_0], [q_0, Z_0, q_1], [q_0, S, q_0],$
$[q_0, S, q_1], [q_1, Z_0, q_1], [q_1, S, q_1], [q_1, Z_0, q_0]$
$[q_1, S, q_0]$

$$|V_N| = 2^2 \times 2 + 1 = 9$$

2- P: $S \to [q_0, Z_0, q_0]$

$S \to [q_0, Z_0, q_1]$

(3)- $[q_0, X, q_1] \to 1$

(4)- $[q_1, X, q_1] \to 1$

(5)- $[q_1, X, q_1] \to \Lambda$

(6)- $[q_1, Z_0, q_1] \to \Lambda$ $\delta(q_0, 0, Z_0) = (q_0, XZ)$

(1)- $[q_0, Z_0, \underline{q_0}] \to 0[q_0, X, \underline{q_0}][\underline{q_0}, Z, \underline{q_0}]$

$[q_0, Z_0, \underline{q_0}] \to 0[q_0, X, \underline{q_1}][\underline{q_1}, Z, \underline{q_0}]$

$[q_0, Z_0, \underline{q_1}] \to 0[q_0, X, \underline{q_0}][\underline{q_0}, Z, \underline{q_1}]$

$[q_0, Z_0, \underline{q_1}] \to 0[q_0, X, \underline{q_1}][\underline{q_1}, Z, \underline{q_1}]$

(2) $[q_0, X, q_0] \to 0[q_0, X, q_0][q_0, X, q_0]$

$[q_0, X, q_0] \to 0[q_0, X, q_1][q_1, X, q_0]$

$[q_0, X, q_1] \to 0[q_0, X, q_0][q_0, X, q_1]$

$[q_0, X, q_1] \to 0[q_0, X, q_1][q_1, X, q_1]$

Ques. $A = (\{q_0, q_1\}, \{a, b\}, \{A, Z_0\}, \delta, q_0, Z_0, \phi)$

(i) $\delta(q_0, a, Z_0) = (q_0, AZ_0)$

(ii) $\delta(q_0, a, A) = (q_0, AA)$

(iii) $\delta(q_0, b, A) = (q_1, A)$

(iv) $\delta(q_1, b, A) = (q_1, A)$

(v) $\delta(q_1, a, A) = (q_1, \Lambda)$

(vi) $\delta(q_1, \Lambda, Z_0) = (q_1, \Lambda)$

Solⁿ. Step-1 - Convert into CFG

Let $G = (V_N, \Sigma, P, S)$ which is equivalent to the given PDA and $L(G) = N(M)$

(a) Constriction of 'V_N' :- $(2^2 * 2) + 1 = 9$

$V_N = S$

$S, [q_0, A, q_0], [q_0, A, q_1], [q_0, Z_0, q_0], [q_0, Z_0, q_1],$
$[q_1, A, q_1], [q_1, A, q_0], [q_1, Z_0, q_1], [q_1, Z_0, q_0].$

(b) $T = \Sigma = \{a, b\}$

(c) P: $S \rightarrow [q_0, Z_0, q_0]$

$S \rightarrow [q_0, Z_0, q_1]$

(d) Transition Rule :-

Acc. to Transition Rule no. 5

(5) $\delta(q_1, a, A) = (q_1, \Lambda)$

$[q_1, A, q_1] \rightarrow a$

(6) $\delta(q_1, \Lambda, Z_0) = (q_1, \Lambda)$

$[q_1, Z_0, q_1] \rightarrow \Lambda$

③ $\delta(q_0, b, A) = (q_1, A)$

$[q_0, A, \underline{q_0}] \rightarrow b[q_1, A, \underline{q_0}]$

$[q_0, A, \underline{q_1}] \rightarrow b[q_1, A, \underline{q_1}]$

④ $\delta(q_1, b, A) = (q_1, A)$

$[q_1, A, \underline{q_0}] \rightarrow b[q_1, A, \underline{q_0}]$

$[q_1, A, \underline{q_1}] \rightarrow b[q_1, A, \underline{q_1}]$

② $\delta(q_0, a, A) = (q_0, AA)$

$[q_0, A, q_0] \rightarrow a[q_0, A, q_0][q_0, A, q_0]$

$[q_0, A, q_1] \rightarrow a[q_0, A, q_1][q_1, A, q_1]$

$[q_0, A, q_0] \rightarrow a[q_0, A, q_1][q_1, A, q_0]$

$[q_0, A, q_1] \rightarrow a[q_0, A, q_0][q_0, A, q_1]$

① $\delta(q_0, a, Z_0) = (q_0, AZ_0)$

$[q_0, Z_0, q_0] \rightarrow a[q_0, A, q_0][q_0, Z_0, q_0]$

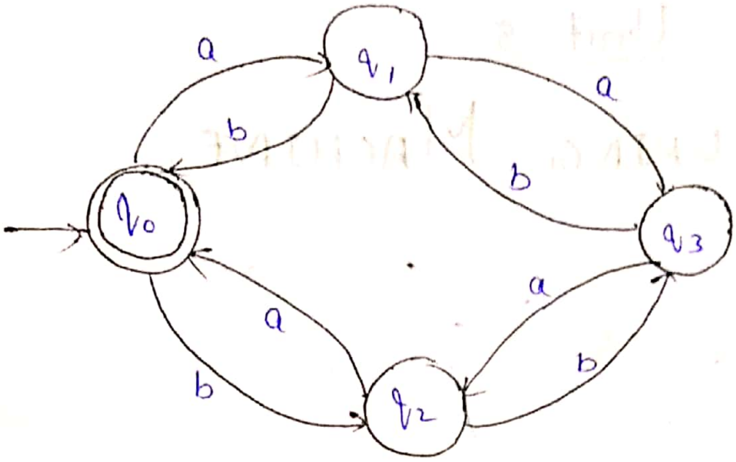
$[q_0, Z_0, q_0] \rightarrow a[q_0, A, q_1][q_1, Z_0, q_0]$

$[q_0, Z_0, q_1] \rightarrow a[q_0, A, q_0][q_0, Z_0, q_1]$

$[q_0, Z_0, q_1] \rightarrow a[q_0, A, q_1][q_1, Z_0, q_1]$

Que. Construct a PDA 'A' accepting the set of all string over a & b with equal no. of a & b.

Soln.- Step 1:- Construct a finite Automaton



Production :- $S \to asb \mid bsa \mid ab \mid ba$

Step 2 :- Convert into GNF

$S \to aSB \mid bSA \mid aB \mid bA$ $\quad B \to b$

$A_1 \to aA_1A_3$ $\quad A \to a$

$A_1 \to bA_1A_2$ $\quad B \to b$

$A_1 \to aA_3$ $\quad A \to a$

$A_1 \to bA_2$

$S \to A_1$

$A \to A_2$

$B \to A_3$

Step 3 :- Transition Rule :-

$(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$A = (\{q_0, q_1, q_f\}, \{a, b\}, \{Z_0, S, A, B\}, \delta, q_0, Z_0, q_f)$

Initial state :-

$\delta(q_0, \Lambda, Z_0) = (q_1, SZ_0)$

$\delta(q_1, a, S) = (q_1, SB) \Rightarrow S \to aSB$

$\delta(q_1, b, S) = (q_1, SA) \Rightarrow S \to bSA$

$\delta(q_1, a, S) = (q_1, B) \Rightarrow S \to aB$

$\delta(q_1, b, S) = (q_1, A) \Rightarrow S \to aA$

$\delta(q_1, a, A) = (q_1, \Lambda) \Rightarrow A \to a$

$\delta(q_1, b, B) = (q_1, \Lambda) \Rightarrow B \to b$

Final state :-

$\delta(q_1, \Lambda, Z_0) = (q_f, \Lambda)$